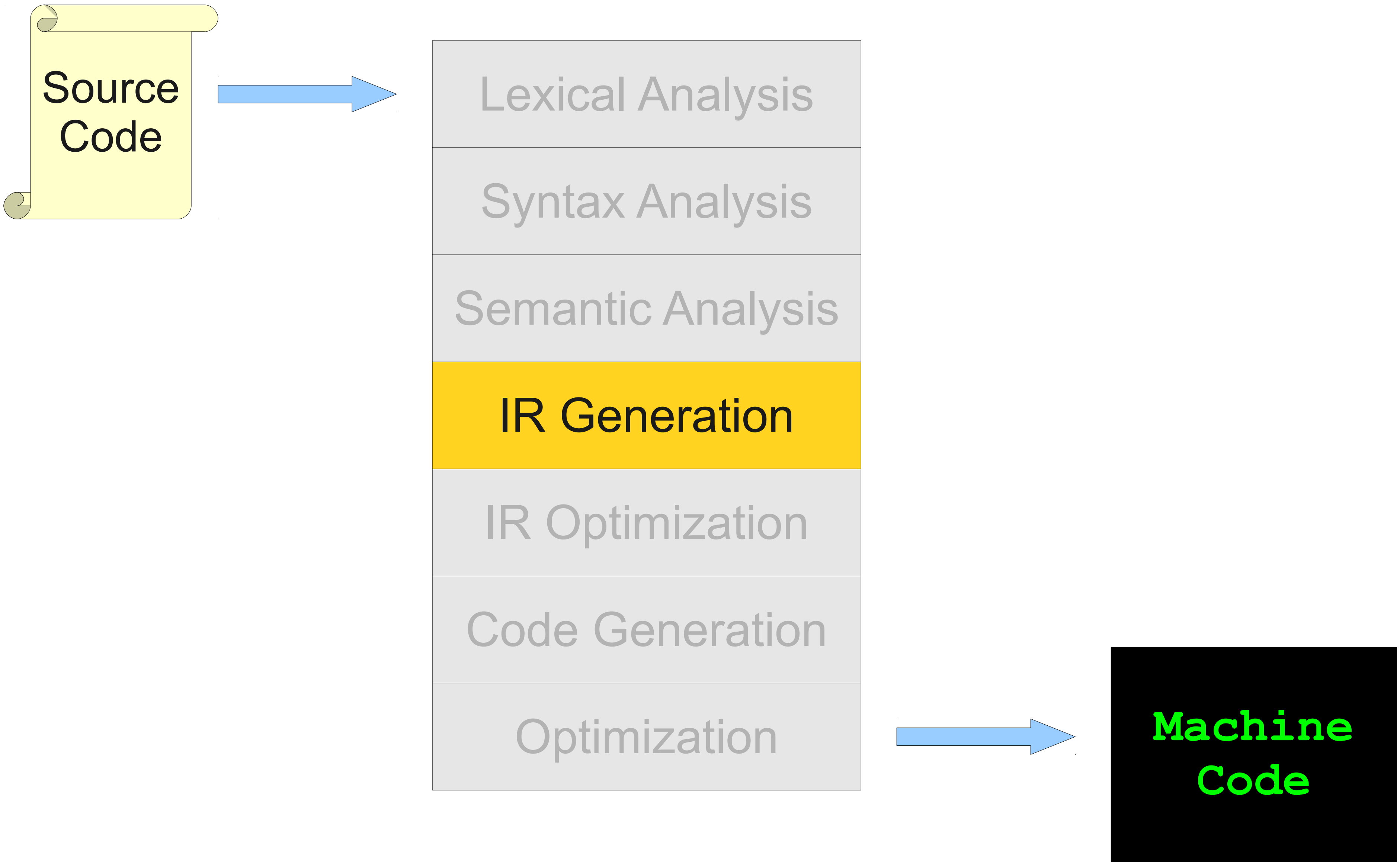
IR Optimization

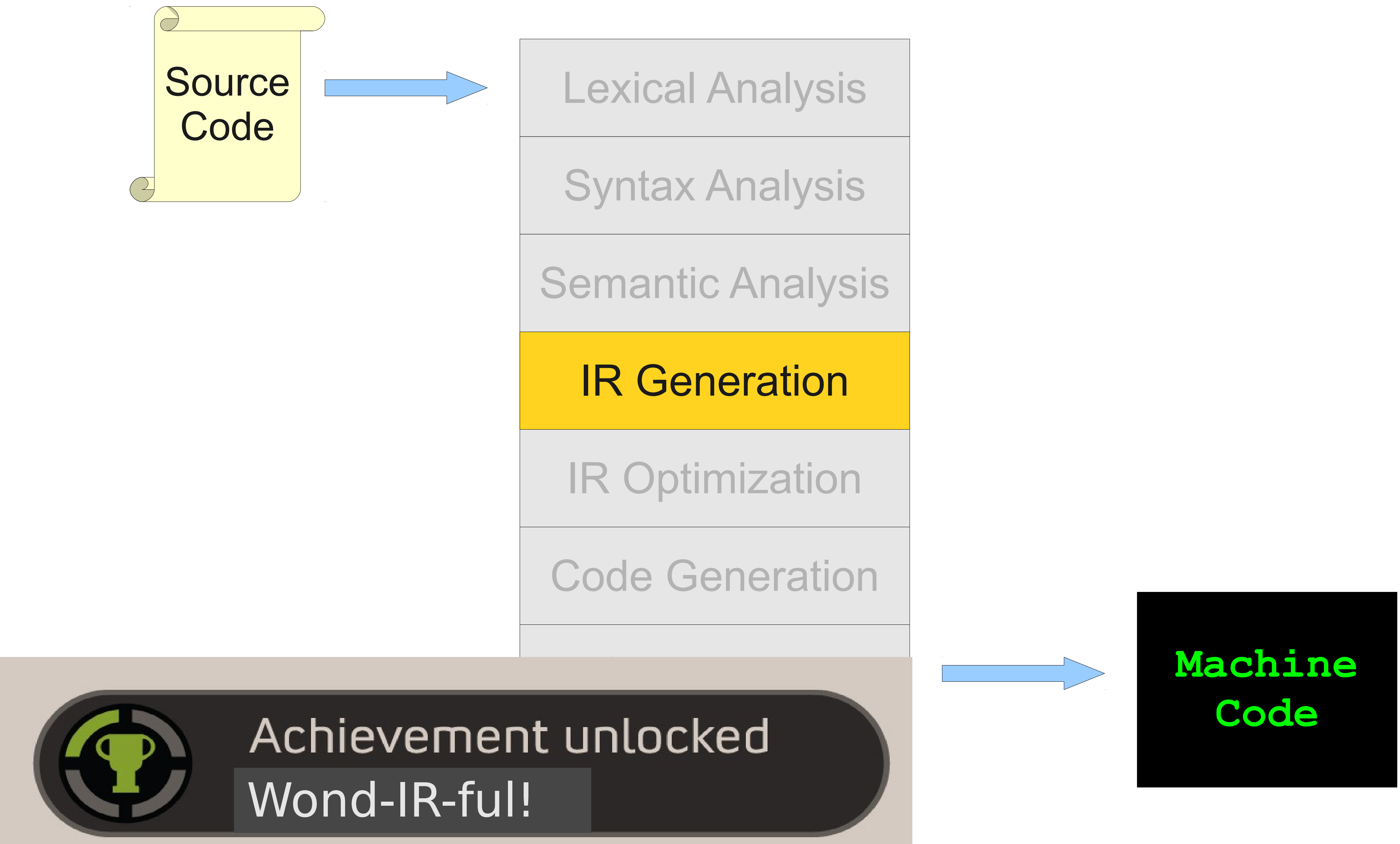
Announcements

- Programming Project 4 due **Wednesday, August 10** at 11:59PM.
- Written Assignment 2 graded.
  - Hard copies returned after lecture.
  - Electronic copies returned by email.
- Programming Assignment 2 grading almost done.
  - We'll email back feedback later today.

Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

Where We Are



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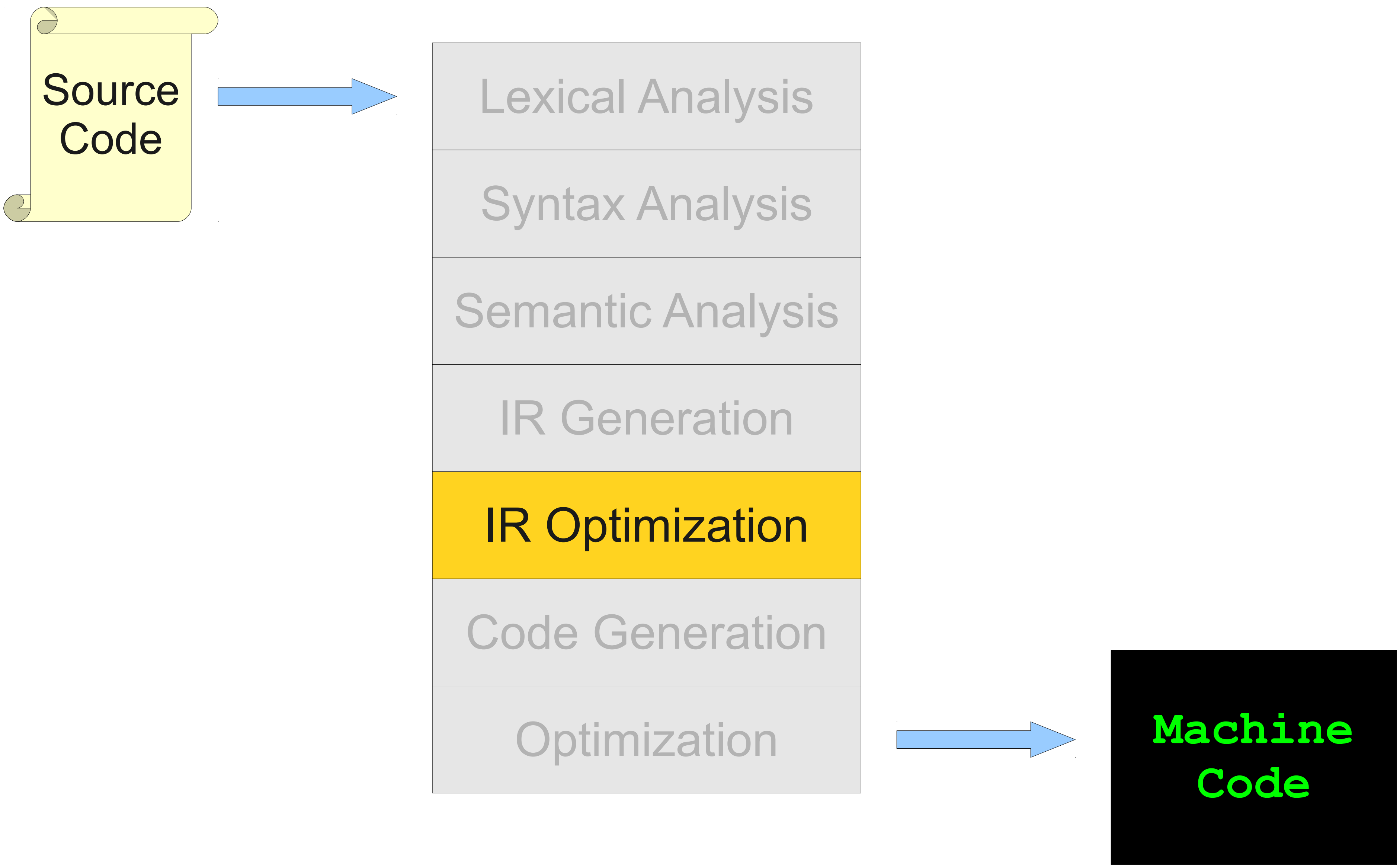
Machine Code
Achievement unlocked
Wond-IR-ful!

Where We Are



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IR Optimization

- **Goal**: Improve the IR generated by the previous step to take better advantage of resources.
- One of the most important and complex parts of any modern compiler.
- A very active area of research.
- There is a whole class (CS243) dedicated to this material.

Sources of Optimization

- In order to optimize our IR, we need to understand why it can be improved in the first place.
- **Reason one:** IR generation introduces redundancy.
  - A naïve translation of high-level language features into IR often introduces subcomputations.
  - Those subcomputations can often be sped up, shared, or eliminated.
- **Reason two:** Programmers are lazy.
  - Code executed inside of a loop can often be factored out of the loop.
  - Language features with side effects often used for purposes other than those side effects.

Optimizations from IR Generation

```
int x;
int y;
bool b1;
bool b2;
bool b3;

b1 = x + x < y
b2 = x + x == y
b3 = x + x > y
```

Optimizations from IR Generation

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int x;
int y;
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```

```
_t0 = x + x;
_t1 = y;
b1 = _t0 < _t1;

_t2 = x + x;
_t3 = y;
b2 = _t2 == _t3;

_t4 = x + x;
_t5 = y;
b3 = _t5 < _t4;
```

Optimizations from IR Generation

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int x;
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_t0 = x + x;
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Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
}
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```
while (x < y + z) {
    x = x - y;
}
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```
_L0:
    _t0 = y + z;
    _t1 = x < _t0;
    IfZ _t1 Goto _L1;
    x = x - y;
    Goto _L0;
_L1:
```

Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
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_L0:
    _t0 = y + z;
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    Goto _L0;
_L1:
```

A Note on Terminology

- The term “optimization” implies looking for an “optimal” piece of code for a program.
- This is, in general, undecidable.
  - e.g. create a program that can be simplified iff some other program halts.
- Our goal will be **IR improvement** rather than **IR optimization**.

The Challenge of Optimization

- A good optimizer
  - Should never change the observable behavior of a program.
  - Should produce IR that is as efficient as possible.
  - Should not take too long to process inputs.
- Unfortunately:
  - Even good optimizers sometimes introduce bugs into code.
  - Optimizers often miss “easy” optimizations due to limitations of their algorithms.
  - Almost all interesting optimizations are NP-hard.

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?

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- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?
- **Runtime** (make the program as fast as possible at the expense of time and power)
- **Memory usage** (generate the smallest possible executable at the expense of time and power)
- **Power consumption** (choose simple instructions at the expense of speed and memory usage)
- Plus a lot more (minimize function calls, reduce use of floating-point hardware, etc.)

IR Optimization vs Code Optimization

- There is not always a clear distinction between what belongs to "IR optimization" versus "code optimization."
- Typically:
  - IR optimizations try to perform simplifications that are valid across all machines.
  - Code optimizations try to improve performance based on the specifics of the machine.
- Some optimizations are somewhere in-between:
  - Replacing `x / 2` with `x * 0.5`

Overview of IR Optimization

- **Formalisms and Terminology** (Today)
  - Control-flow graphs.
  - Basic blocks.
- **Local optimizations** (Today)
  - Speeding up small pieces of a function.
- **Global optimizations** (Monday)
  - Speeding up functions as a whole.
- **The dataflow framework** (Monday/Wednesday)
  - Defining and implementing a wide class of optimizations.
- **Lazy code motion** (Wednesday)
  - An extremely powerful IR optimization used in many compilers.

Formalisms and Terminology

Analyzing a Program

- In order to optimize a program, the compiler has to be able to reason about the properties of that program.
- An analysis is called **sound** if it never asserts an incorrect fact about a program.
- All the analyses we will discuss in this class are sound.
  - *(Why?)*

Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
else
    x = 42;

Print(x);
```

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int x;
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if (y < 5)
    x = 137;
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Print(x);
```

"At this point in the program, **x** holds some integer value."

Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
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    x = 42;

Print(x);
```

"At this point in the program, **x** is either 137 or 42"

Soundness

```
int x;
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if (y < 5)
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else
    x = 42;

Print(x);
```

“At this point in the program, **x** is 137”

Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
else
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Print(x);
```

"At this point in the program, **`x`** is either 137, 42, or 271"

Semantics-Preserving Optimizations

- An optimization is **semantics-preserving** if it does not alter the semantics of the original program.
- Examples:
  - Eliminating unnecessary temporary variables.
  - Computing values that are known statically at compile-time instead of runtime.
  - Evaluating constant expressions outside of a loop instead of inside.
- Non-examples:
  - Replacing bubble sort with quicksort.
- The optimizations we will consider in this class are all semantics-preserving.

A Formalism for IR Optimization

- Every phase of the compiler uses some new abstraction:
  - Scanning uses regular expressions.
  - Parsing uses CFGs.
  - Semantic analysis uses proof systems and symbol tables.
  - IR generation uses ASTs.
- In optimization, we need a formalism that captures the structure of a program in a way amenable to optimization.

Visualizing IR

```
main:
    BeginFunc 40;
    _tmp0 = LCall _ReadInteger;
    a = _tmp0;
    _tmp1 = LCall _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    PushParam a;
    LCall _PrintInt;
    PopParams 4;
    EndFunc;
```

Visualizing IR

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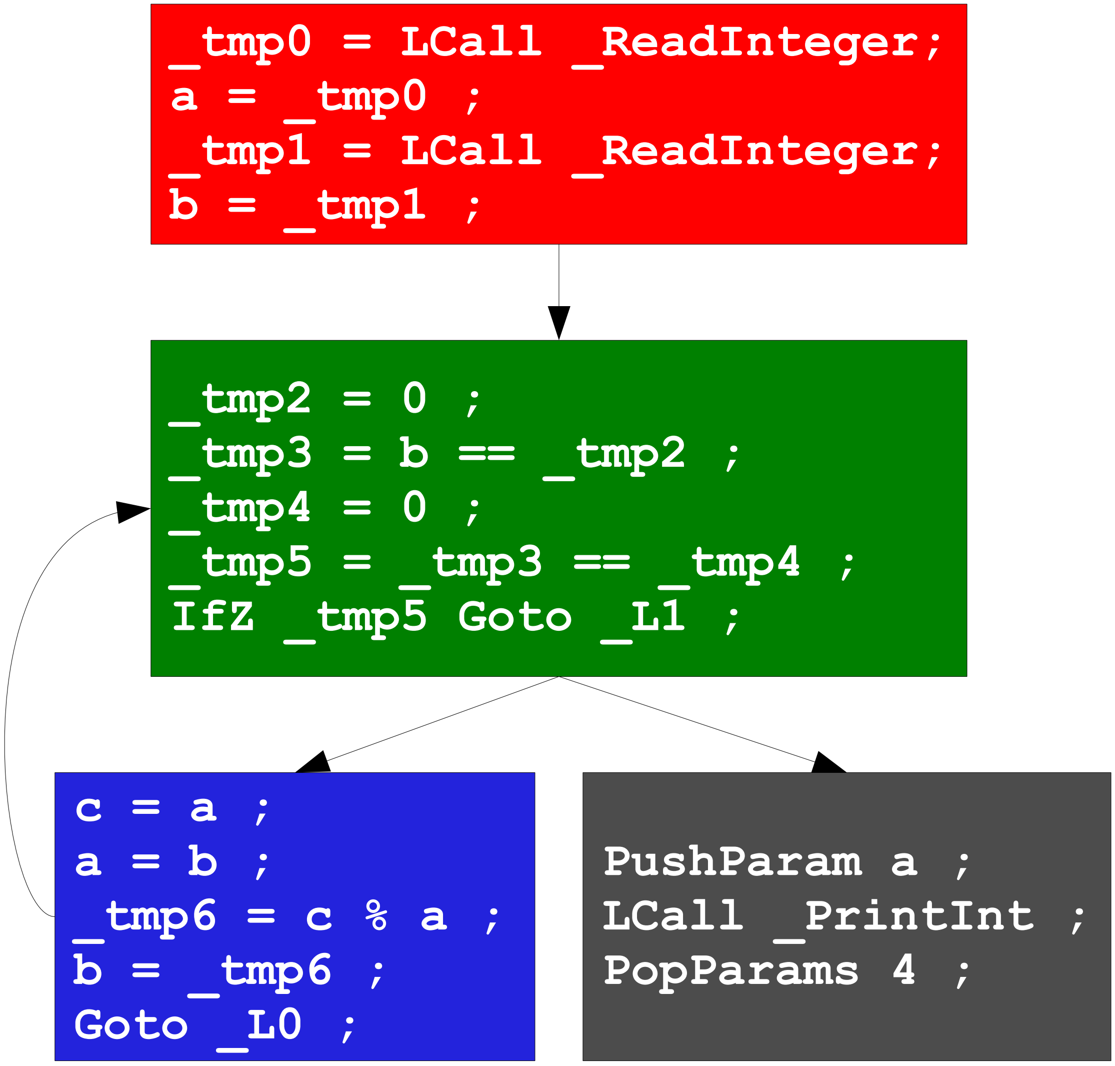

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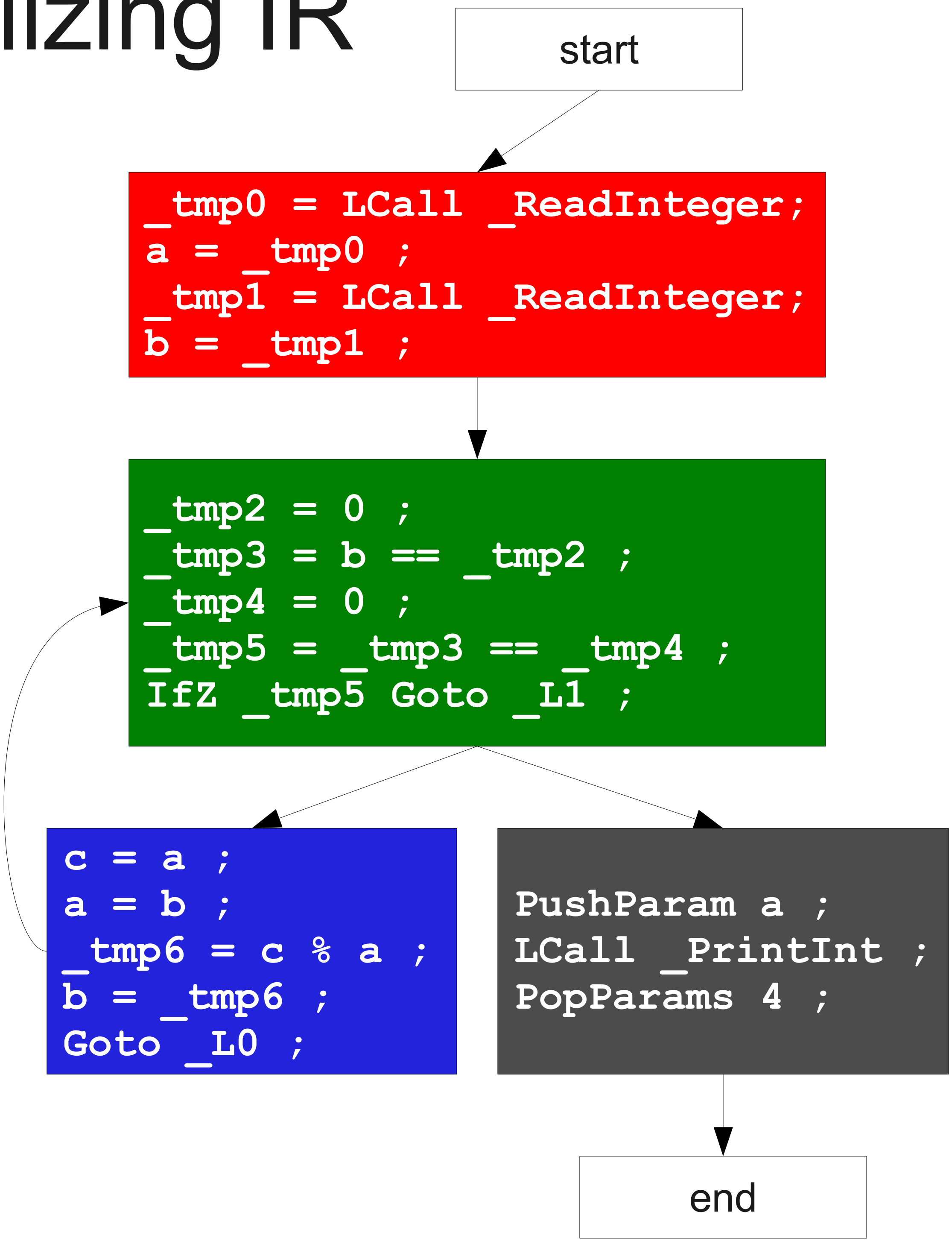


Basic Blocks

- A **basic block** is a sequence of IR instructions where
  - There is exactly one spot where control enters the sequence, which must be at the start of the sequence.
  - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence.
- Informally, a sequence of instructions that always execute as a group.

Control-Flow Graphs

- A **control-flow graph** (CFG) is a graph of the basic blocks in a function.
  - The term CFG is overloaded – from here on out, we'll mean “control-flow graph” and not “context-free grammar.”
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block.
- There is a dedicated node for the start and end of a function.

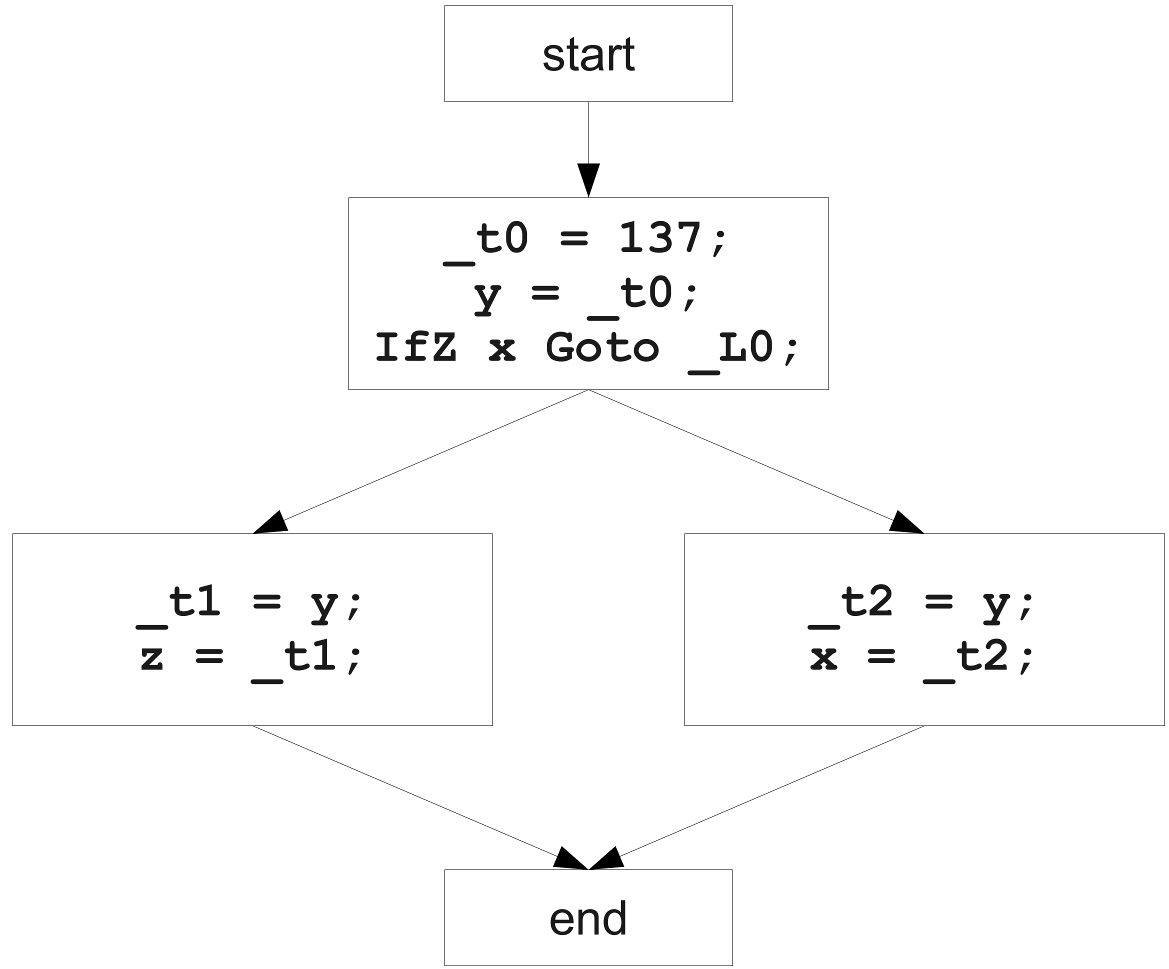
Types of Optimizations

- An optimization is **local** if it works on just a single basic block.
- An optimization is **global** if it works on an entire control-flow graph.
- An optimization is **interprocedural** if it works across the control-flow graphs of multiple functions.
  - We won't talk about this in this course.

Local Optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



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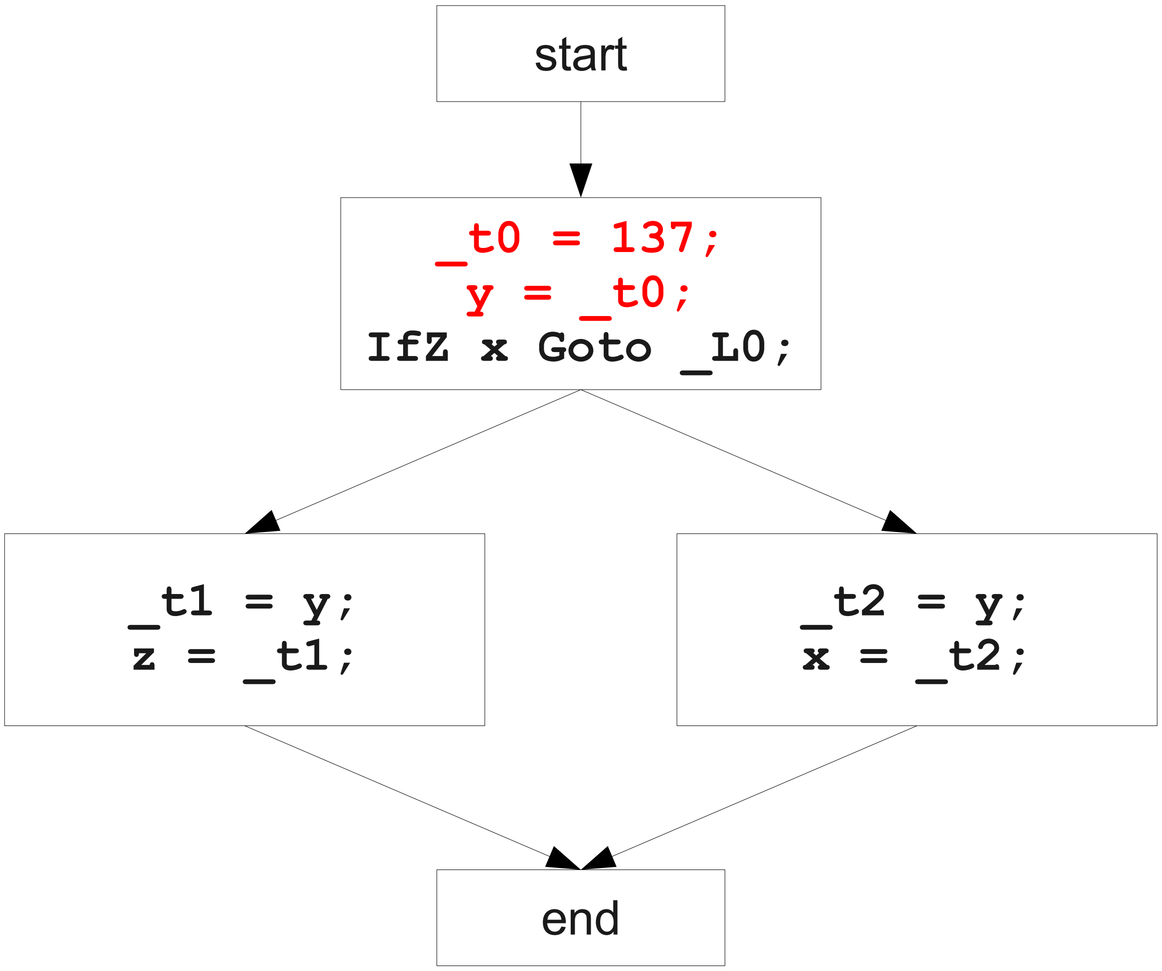

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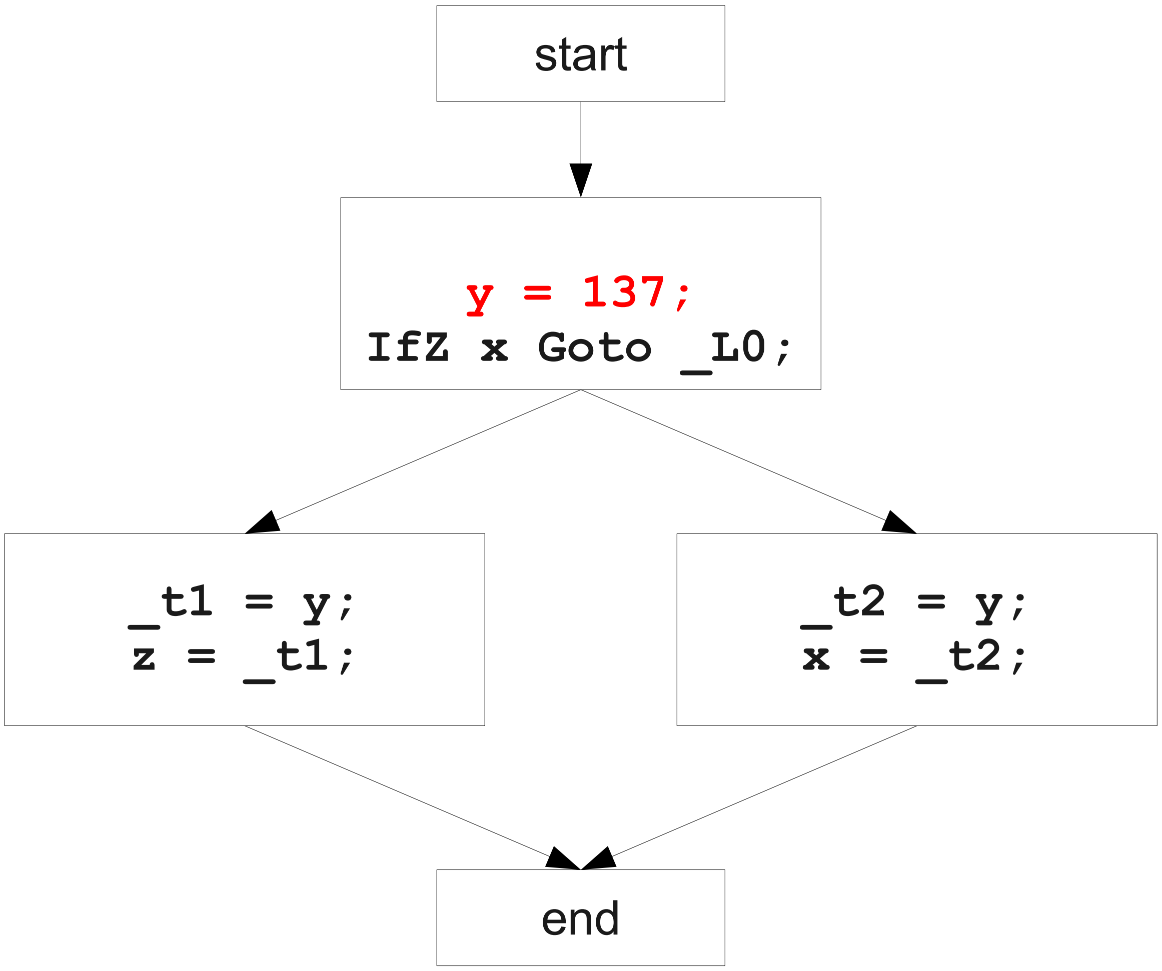

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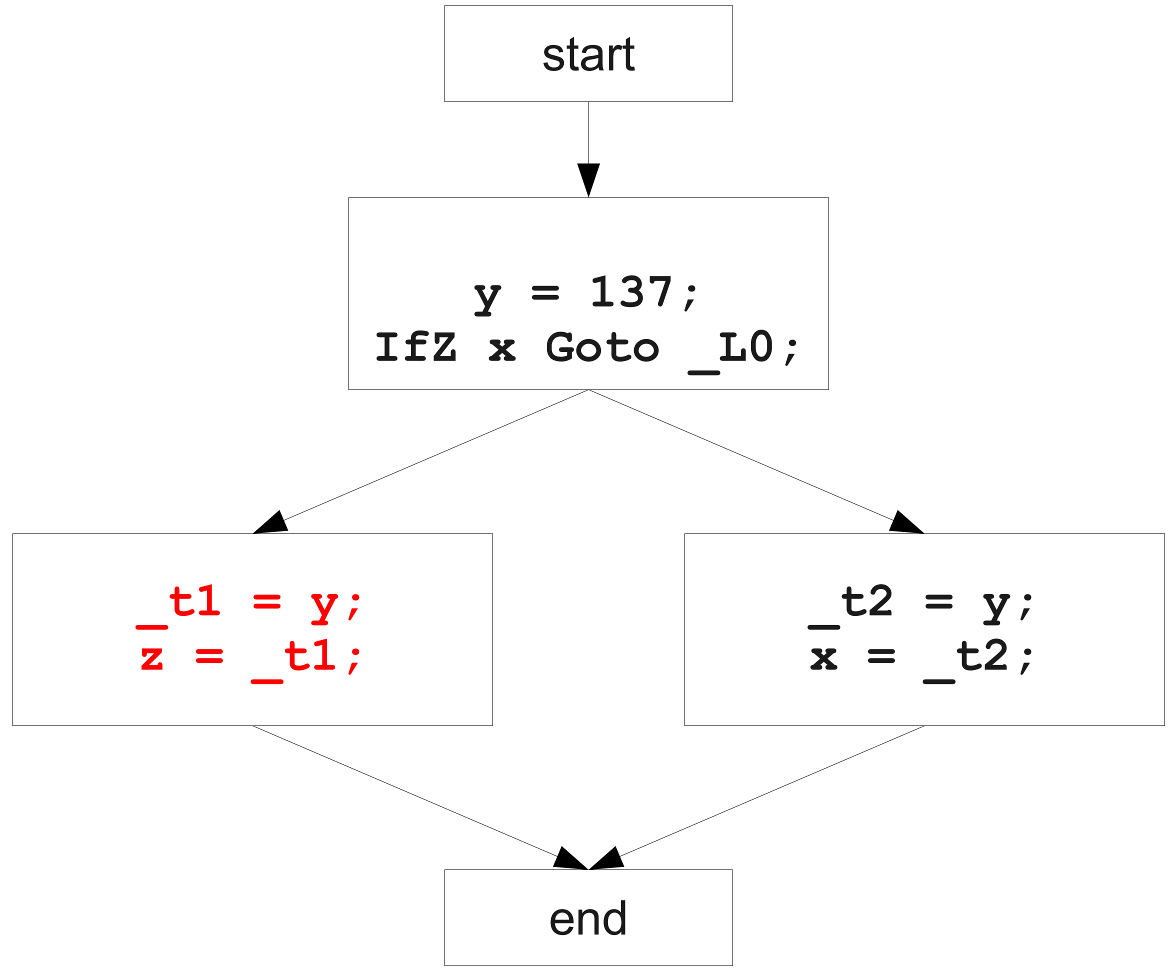

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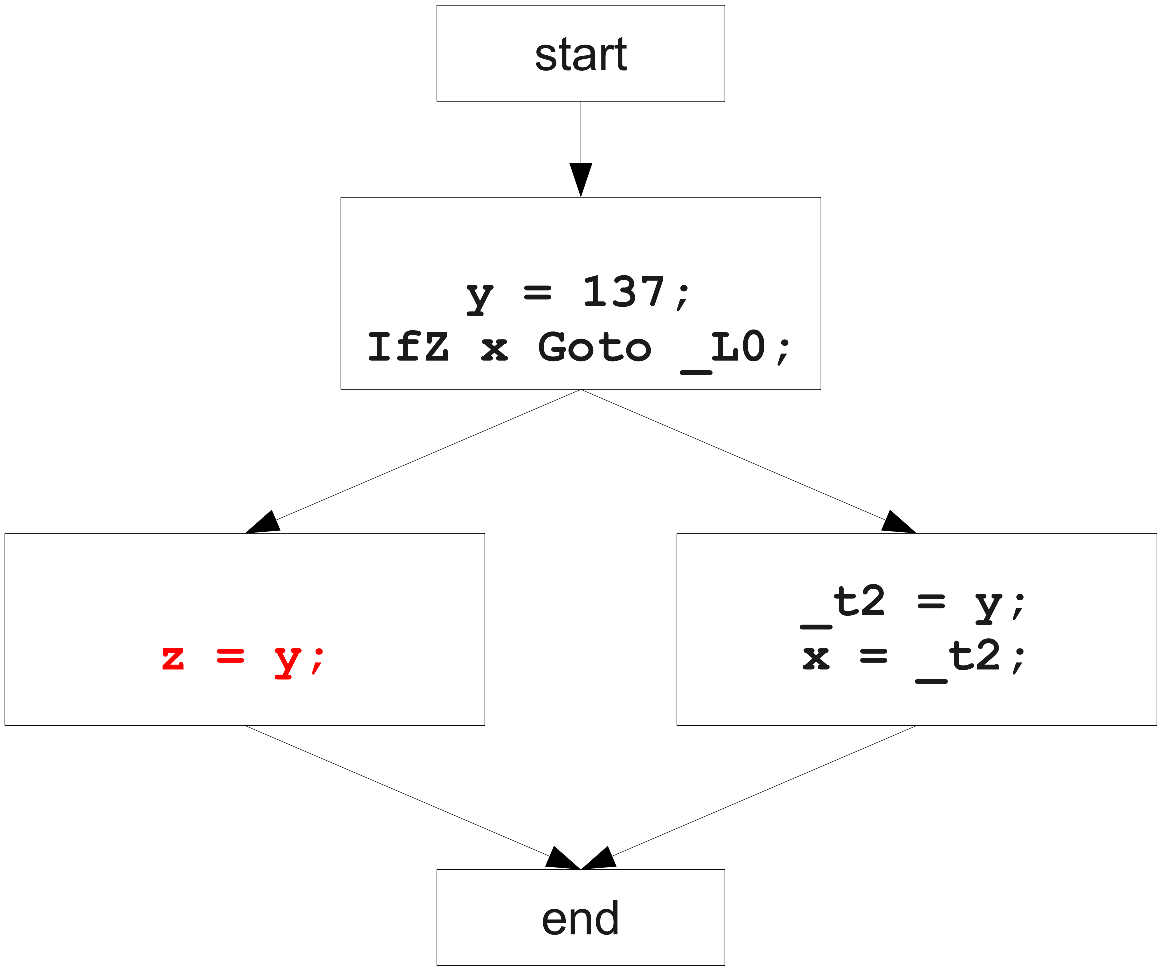

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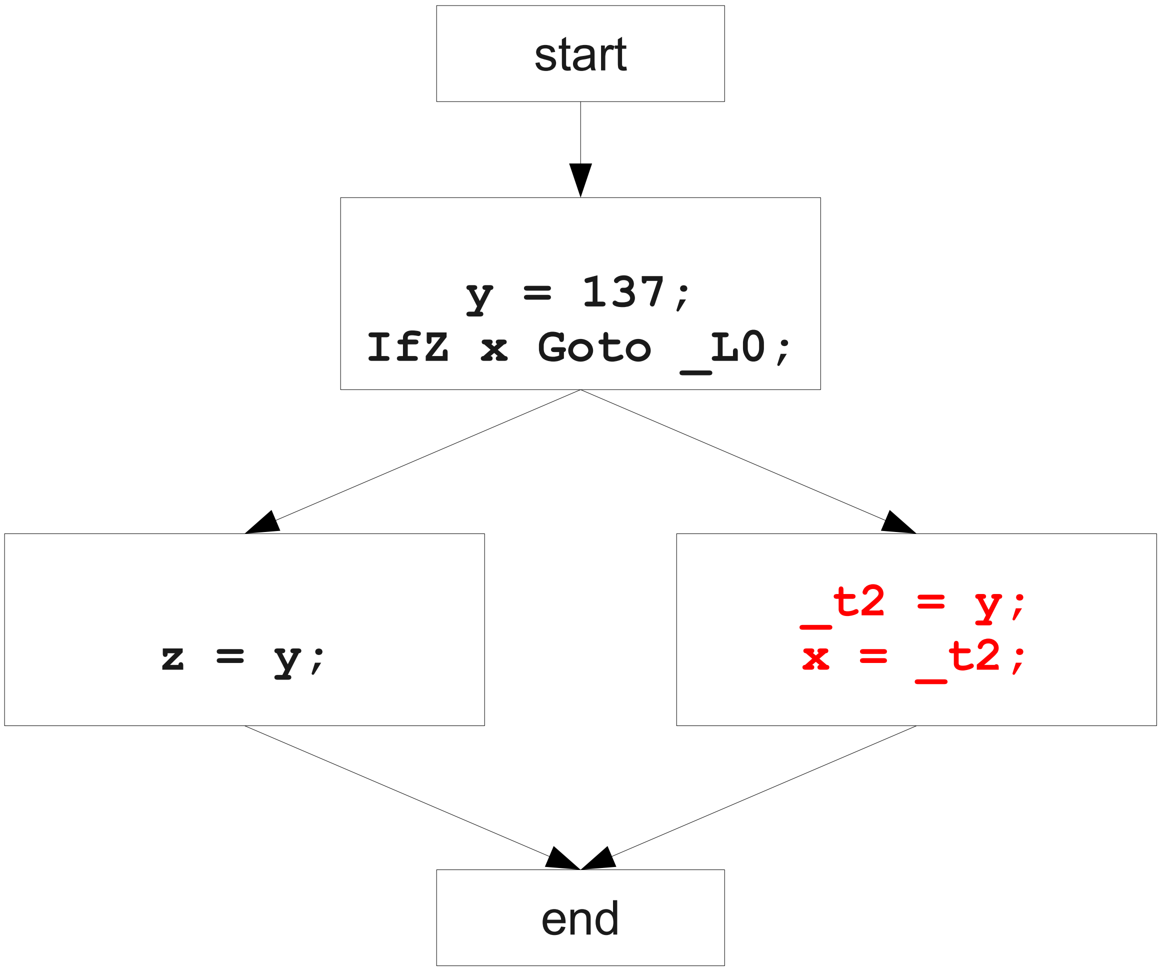

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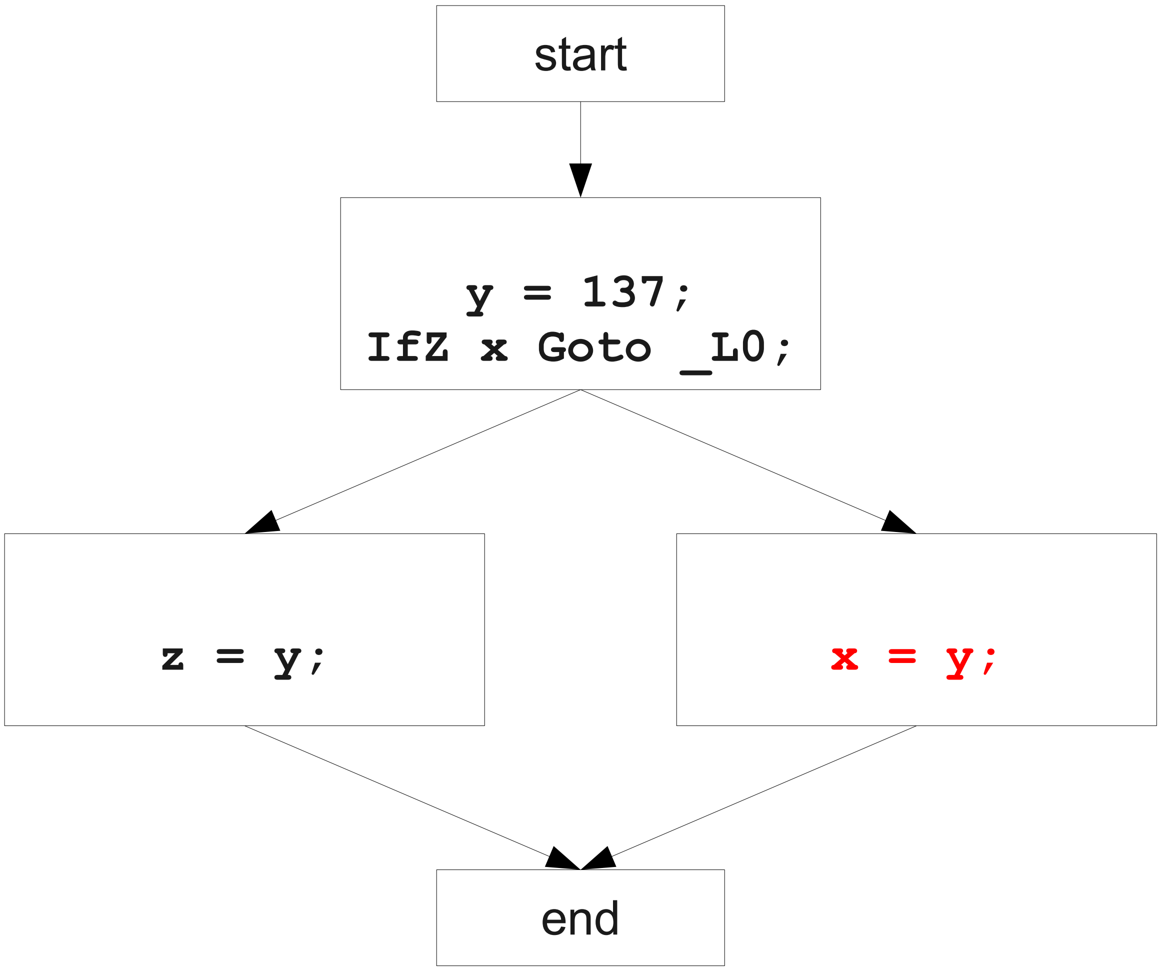

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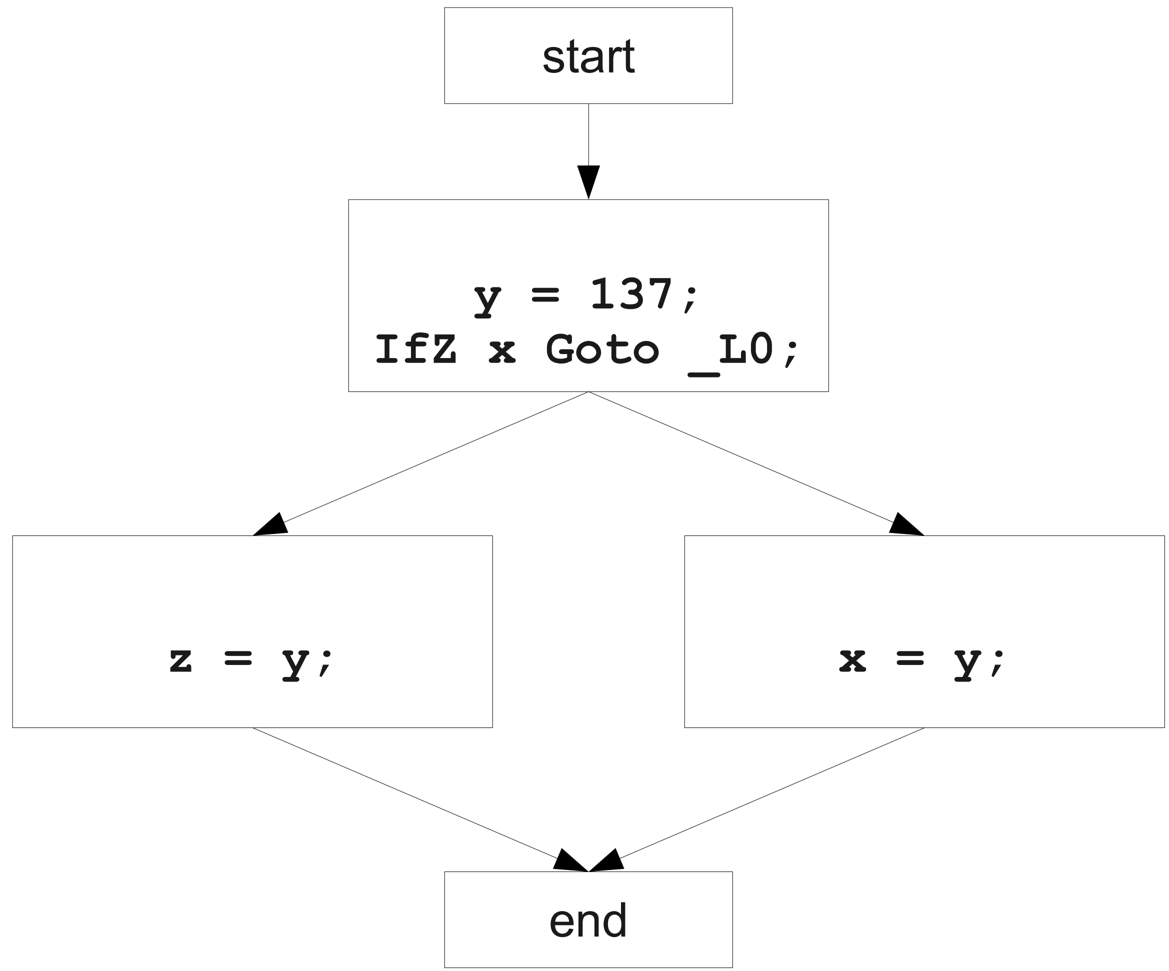

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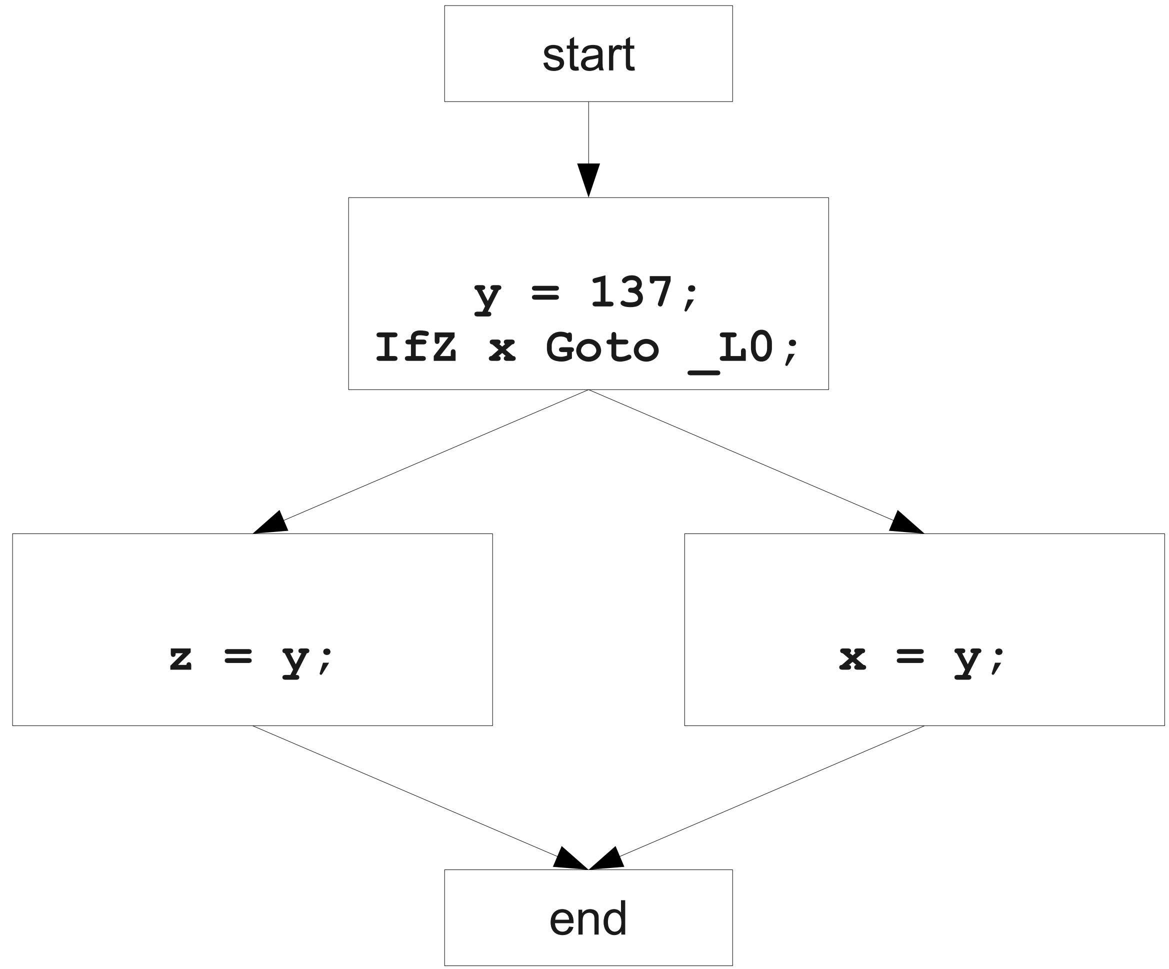

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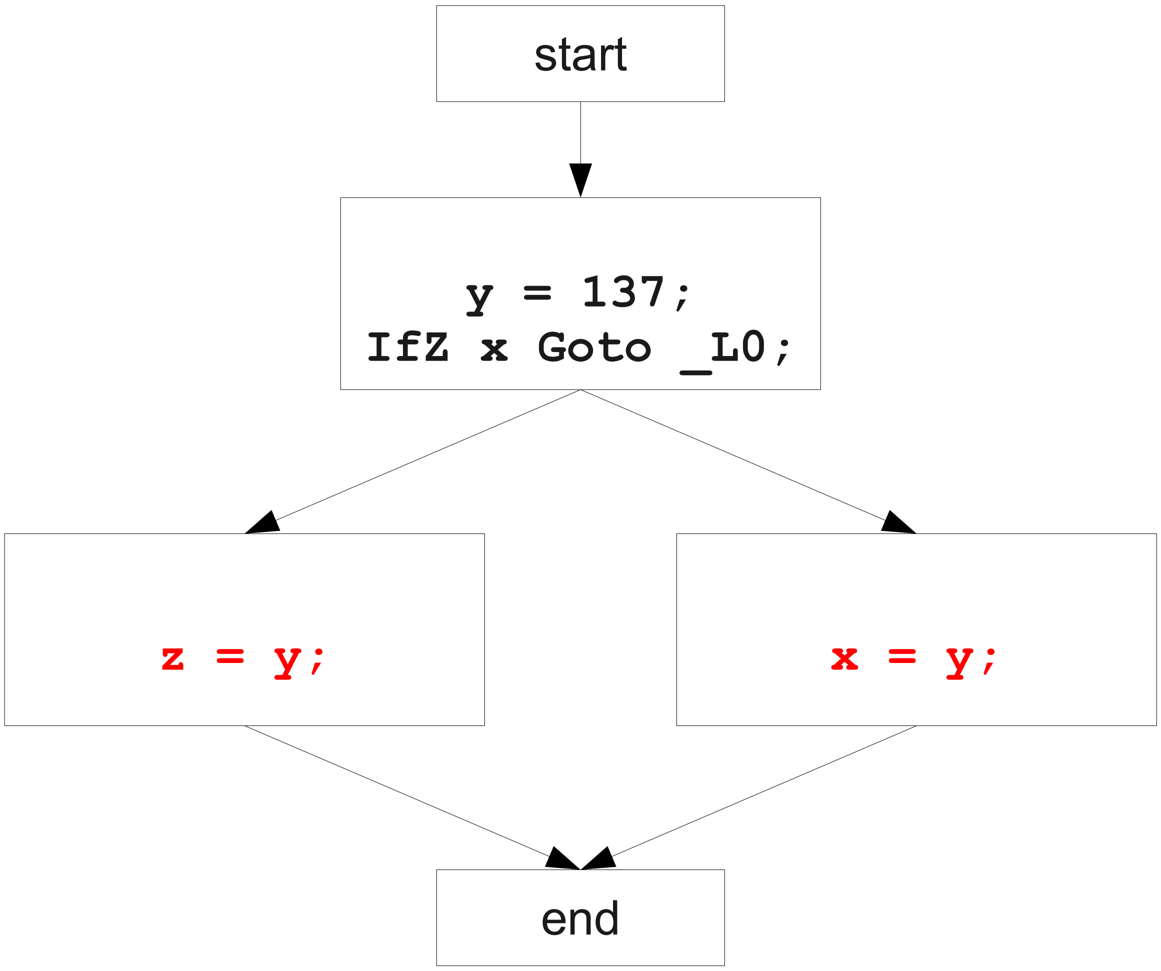

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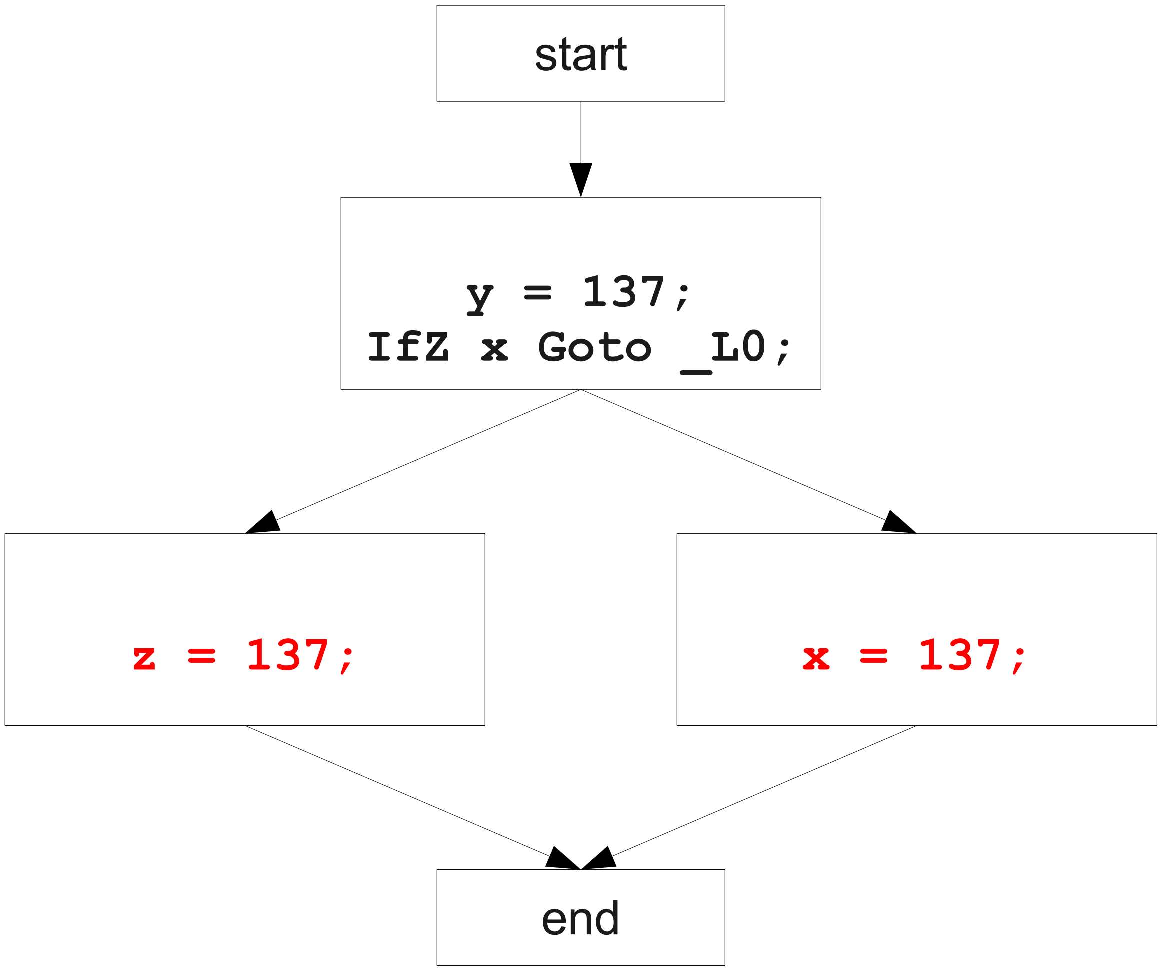

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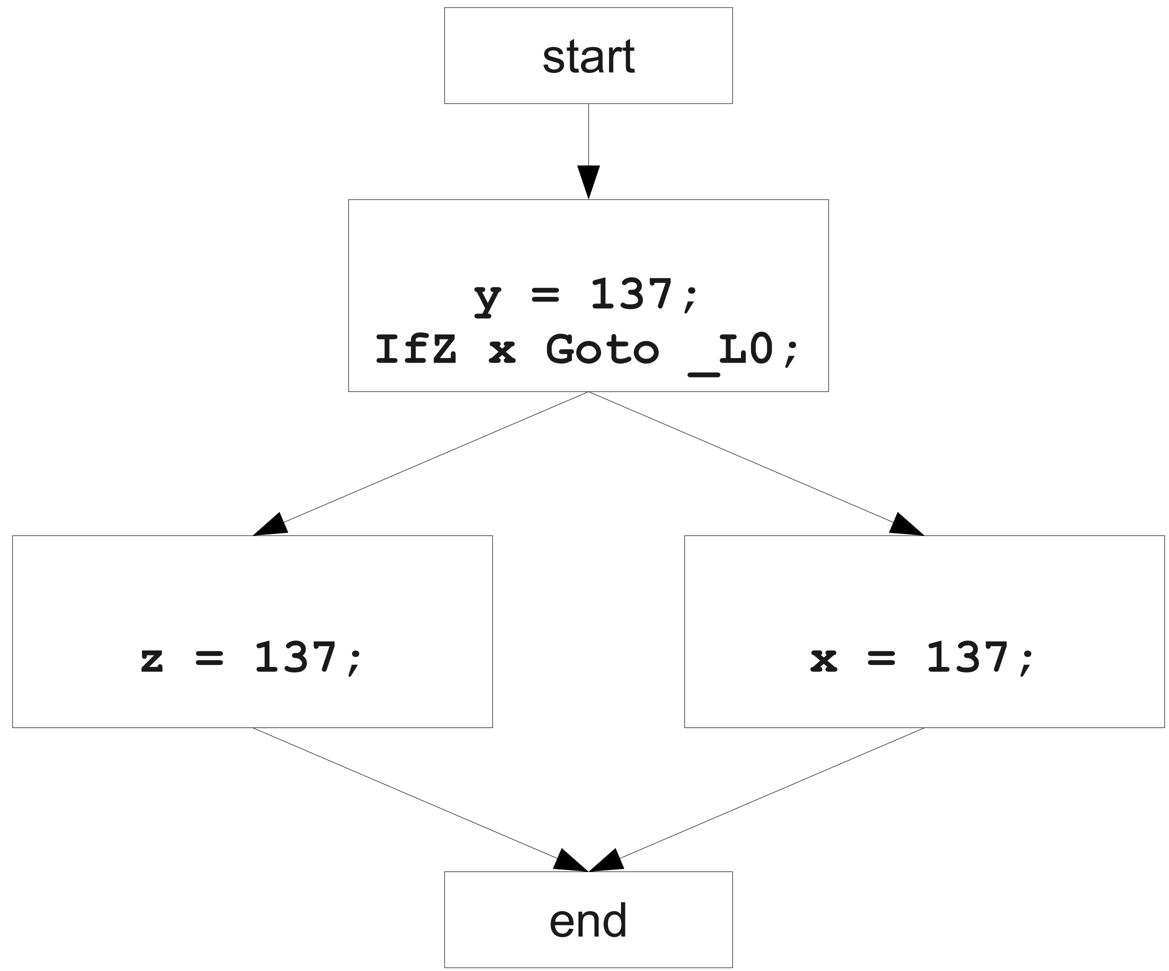

Local Optimizations

Common Subexpression Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

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_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = a + b ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
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int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

Common Subexpression Elimination

- If we have two variable assignments

  $\mathbf{v_1 = a \ op \ b}$

  ...

  $\mathbf{v_2 = a \ op \ b}$

  and the values of $\mathbf{v_1}$, **a**, and **b** have not changed between the assignments, rewrite the code as

  $\mathbf{v_1 = a \ op \ b}$

  ...

  $\mathbf{v_2 = v_1}$

- Eliminates useless recalculation.
- Paves the way for later optimizations.

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
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```

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Copy Propagation

```
Object x;
int a;
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Object x;
int a;
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Object x;
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_tmp5 = c ;
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_tmp7 = *(_tmp6) ;
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ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
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x = new Object;
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_tmp5 = c ;
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ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
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x = new Object;
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_tmp0 = 4 ;
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_tmp5 = c ;
_tmp6 = *(_tmp1) ;
_tmp7 = *(_tmp6) ;
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ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

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Object x;
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_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp6) ;
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ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

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Object x;
int a;
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x = new Object;
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_tmp0 = 4 ;
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_tmp5 = c ;
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Copy Propagation

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Object x;
int a;
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x = new Object;
a = 4;
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_tmp0 = 4 ;
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_tmp5 = c ;
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Copy Propagation

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Object x;
int a;
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x = new Object;
a = 4;
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_tmp0 = 4 ;
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Copy Propagation

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Object x;
int a;
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int c;

x = new Object;
a = 4;
c = a + b;
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```

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_tmp0 = 4 ;
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Object x;
int a;
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_tmp0 = 4 ;
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Object x;
int a;
int b;
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x = new Object;
a = 4;
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_tmp0 = 4 ;
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_tmp1 = LCall _Alloc ;
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x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
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PushParam c ;
PushParam _tmp1 ;
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Copy Propagation

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Object x;
int a;
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x = new Object;
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_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
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```

Copy Propagation

```
Object x;
int a;
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x = new Object;
a = 4;
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_tmp0 = 4 ;
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c = _tmp4 ;
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_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
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Copy Propagation

```
Object x;
int a;
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x = new Object;
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c = a + b;
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_tmp0 = 4 ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
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a = 4 ;
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c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

- If we have a variable assignment

  $v_1 = v_2$

  then as long as $\mathbf{v_1}$ and $\mathbf{v}_2$ are not reassigned, we can rewrite expressions of the form

  $a = \ldots v_1 \ldots$

  as

  $a = \ldots v_2 \ldots$

  provided that such a rewrite is legal.
- This will help immensely later on, as you'll see.

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
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x = new Object;
a = 4;
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
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_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
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Dead Code Elimination

```
Object x;
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x = new Object;
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_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
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Object x;
int a;
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c = _tmp4 ;
_tmp5 = _tmp4 ;
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Object x;
int a;
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_tmp0 = 4 ;
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Object x;
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_tmp0 = 4 ;
PushParam _tmp0 ;
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a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
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Dead Code Elimination

```
Object x;
int a;
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x = new Object;
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_tmp0 = 4 ;
PushParam _tmp0 ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
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Dead Code Elimination

```
Object x;
int a;
int b;
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x = new Object;
a = 4;
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
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x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
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Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;


_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;


_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;



_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

- An assignment to a variable **v** is called **dead** if the value of that assignment is never read anywhere.
- **Dead code elimination** removes dead assignments from IR.
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned.

For Comparison

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = a + b ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
_tmp4 = _tmp0 + b ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Applying Local Optimizations

- The different optimizations we've seen so far all take care of just a small piece of the optimization.
  - Common subexpression elimination eliminates unnecessary statements.
  - Copy propagation helps identify dead code.
  - Dead code elimination removes statements that are no longer needed.
- To get maximum effect, we may have to apply these optimizations numerous times.

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Other Types of Local Optimization

- **Arithmetic Simplification**
  - Replace "hard" operations with easier ones.
  - e.g. rewrite `x = 4 * a;` as `x = a << 2;`
- **Constant Folding**
  - Evaluate expressions at compile-time if they have a constant value.
  - e.g. rewrite `x = 4 * 5;` as `x = 20;`.

Implementing Local Optimization

Optimizations and Analyses

- Most optimizations are only possible given some analysis of the program's behavior.
- In order to implement an optimization, we will talk about the corresponding program analyses.

Available Expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the **available expressions** in a program.
- An expression is called **available** if some variable in the program holds the value of that expression.
- In common subexpression elimination, we replace an available expression by the variable holding its value.
- In copy propagation, we replace the use of a variable by the available expression it holds.

Finding Available Expressions

- Initially, no expressions are available.
- Whenever we execute a statement **a** = **b** + **c**:
  - Any expression holding **a** is invalidated.
  - The expression **a** = **b** + **c** becomes available.
- **Idea**: Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable.

Available Expressions

```
a = b;
c = b;
d = a + b;
e = a + b;
d = b;
f = a + b;
```

Available Expressions

```
{ }
a = b;

c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = e;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

Live Variables

- The analysis corresponding to dead code elimination is called **liveness analysis**.
- A variable is **live** at a point in a program if later in the program its value will be read before it is written to again.
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables.

Computing Live Variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in **reverse order**.
- Initially, some small set of values are known to be live (which ones depends on the particular program).
- When we see the statement **a** = **b** + **c**:
  - Just before the statement, **a** is not alive, since its value is about to be overwritten.
  - Just before the statement, both **b** and **c** are alive, since we're about to read their values.
  - *(what if we have* **a** = **a** + **b***?)*

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }

{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
a = b;


d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;


d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;


d = a + b;

e = d;

d = a;
{ b, d }
```

Liveness Analysis II

```
a = b;



d = a + b;

e = d;
{ a, b }
d = a;
{ b, d }
```

Liveness Analysis II

```
a = b;



d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Liveness Analysis II

```
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Liveness Analysis II

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }

{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
a = b;

d = a + b;

d = a;
```

Liveness Analysis III

```
a = b;


d = a + b;


d = a;
```

Liveness Analysis III

```
a = b;


d = a + b;


d = a;
{b, d}
```

Liveness Analysis III

```
a = b;


d = a + b;

{a, b}

d = a;
{b, d}
```

Liveness Analysis III

```
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Liveness Analysis III

```
    {b}
  a = b;

  {a, b}

d = a + b;

  {a, b}

  d = a;
  {b, d}
```

Dead Code Elimination

```
  {b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}



{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
a = b;
```

```
d = a;
```

A Combined Algorithm

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
{b, d}
```

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;
{a, b}
d = a;


{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;
{a, b}
d = a;


{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;

c = a;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;



{a, b}
d = a;


{b, d}
```

A Combined Algorithm

```
a = b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
{b}
a = b;

{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;
```

```
d = a;
```

Properties of Local Analysis

- The only way to find out what a program will actually do is to run it.
- Problems:
  - The program might not terminate.
  - The program might have some behavior we didn't see when we ran it on a particular input.
- However, this is **not** a problem inside a basic block.
  - Basic blocks contain no loops.
  - There is only one path through the basic block.

Another View of Local Optimization

- In local optimization, we want to reason about some property of the runtime behavior of the program.
- We know that we can run the code in a basic block and guarantee termination.
- Could we run the program and just watch what happens?
- **Idea**: Redefine the semantics of our programming language to give us information about our analysis.

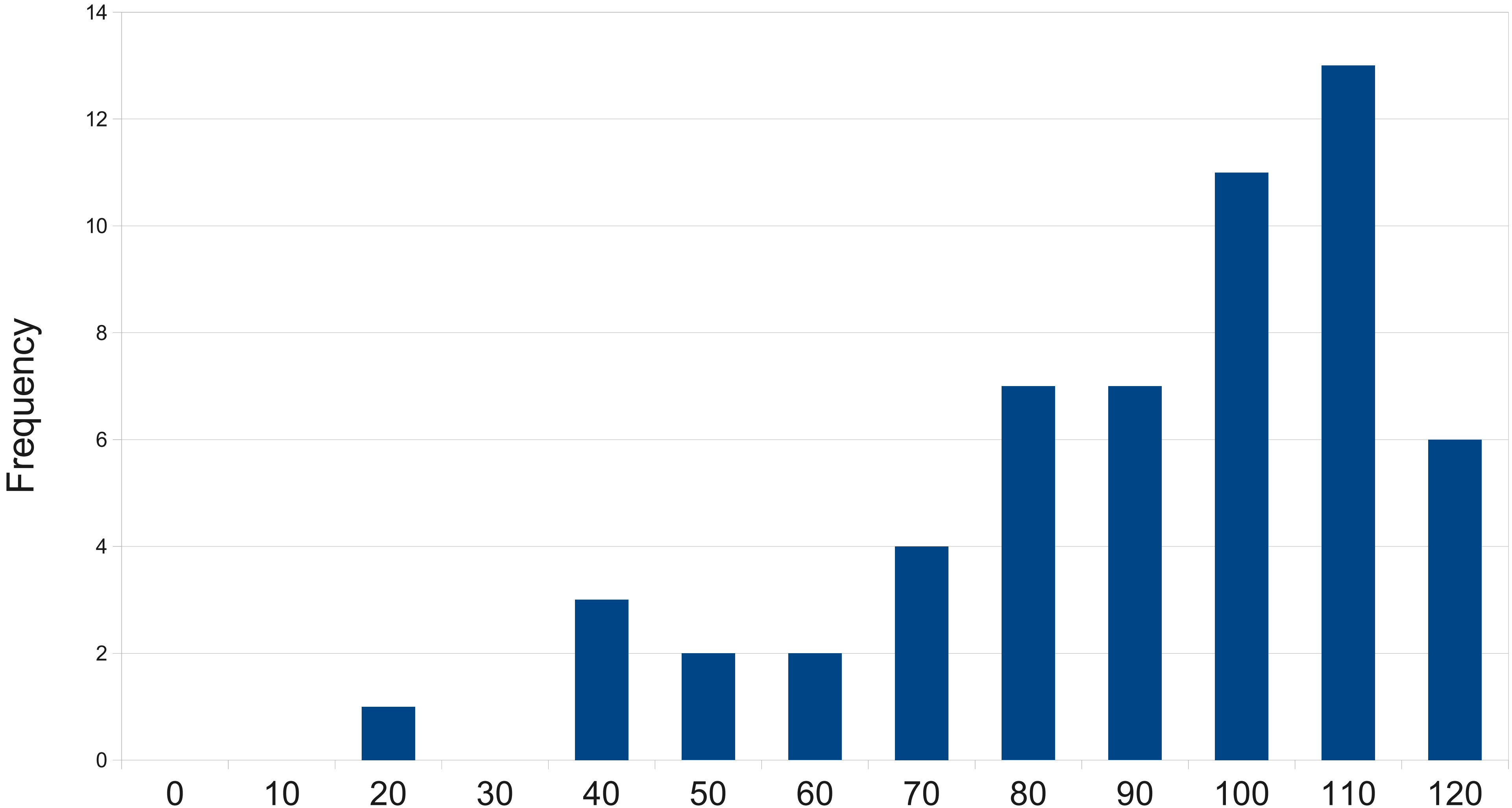
Assigning New Semantics

- Example: Available Expressions
- Redefine the statement **a = b + c** to mean "**a** now holds the value of **b** + **c**, and any variable holding the value **a** is now invalid."
- Run the program assuming these new semantics.
- Treat the compiler as an **interpreter for these new semantics.**

Next Time

- **Global optimization**
  - Optimizing across basic blocks.
  - Meet operators and the dataflow framework.

Written Assignment 2 Scores



Mean: **86 / 120**
Median: **93.5 / 120**
Stdev: **23**